Implicit Differentiation

1) $x = \sec y$

$1 = \sec y \tan y \cdot y'$

$y' = \dfrac{1}{\sec y \tan y}$

$y' = \cos y \cot y$

2) $2x^2 - y^2 = 1$

a) $4x - 2y \cdot \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{2x}{y}$

b) $\dfrac{d^2y}{dx^2} = \dfrac{2y - 2xy'}{y^2}$

$= \dfrac{2y - 2x\left(\frac{2x}{y}\right)}{y^2}$

$= \dfrac{2y - \frac{4x^2}{y}}{y^2} = \dfrac{\frac{2y^2 - 4x^2}{y}}{y^2}$

$\boxed{= \dfrac{2y^2 - 4x^2}{y^3}}$

3) $y^2 = 9x^2 + 4x$

a) $2y\dfrac{dy}{dx} = 18x + 4$

$\dfrac{dy}{dx} = \dfrac{9x+2}{y}$

b) $\dfrac{d^2y}{dx^2} = \dfrac{9y - (9x+2)\left(\frac{dy}{dx}\right)}{y^2}$

$= \dfrac{9y - \frac{(9x+2)^2}{y}}{y^2}$

$= \dfrac{9y^2 - (9x+2)^2}{y^3}$

4) $x^2 - 4xy + y^2 = -6$

$2x - 4x\dfrac{dy}{dx} - 4y + 2y\dfrac{dy}{dx} = 0$

$-4x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 4y - 2x$

$(2y - 4x)\dfrac{dy}{dx} = 4y - 2x$

$\dfrac{dy}{dx} = \dfrac{4y-2x}{2y-4x} = \dfrac{2y-x}{y-2x}$

5) $x^2 + y^2 = 26$

point

$1 + y^2 = 26$

$y^2 = 25$

$y = \pm 5$

$(-1, 5)\ (-1, -5)$

slope

$2x + 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{x}{y}$

$(-1, 5)$

$\dfrac{dy}{dx} = \dfrac{1}{5}$

$(-1, -5)$

$\dfrac{dy}{dx} = -\dfrac{1}{5}$

$\boxed{y - 5 = \frac{1}{5}(x+1)}$

$\boxed{y + 5 = -\frac{1}{5}(x+1)}$

6) $(x-3)^2+(y-4)^2=5$

$2(x-3)+2(y-4)\frac{dy}{dx}=0$

At (5,5):

$2(2)+2\frac{dy}{dx}=0$

$\boxed{\frac{dy}{dx}=-2}$

7) $x^2y^2=16$

$x^2(2y\frac{dy}{dx})+2xy^2=0$

At (−1, 4):

$8\frac{dy}{dx}-32=0$

$\frac{dy}{dx}=4$

T: $y-4=4(x+1)$

N: $y-4=-\frac{1}{4}(x+1)$